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# POLAR SYMMETRY IN THREE-DIMENSIONAL ANALYSIS OF LAMINATES WITH ANGLE PLIES



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## SCOPE

This paper discusses an improved procedure for the 3-D finite-element analysis of an angle-ply laminate with a circular hole. The procedure exploits polar symmetry and is the basis for a new finite-element computer code. For the broad class of laminates that contain angle plies, this new code requires only one-half as much computing capacity as conventional codes. Because previous 3-D analyses have been limited by computer capacity, the new code will improve the analysis of laminates with angle plies.

## FINITE-ELEMENT ANALYSES

To date, only Dana and Barker [1] have presented a complete 3-D finite-element analysis of an angle-ply laminate

with a hole. They modeled one-half of the laminate thickness, exploiting midplane symmetry of their $(\mp 45°)_s$ laminate. But because the 3-D stresses did not possess the usual inplane symmetry, they modeled the entire plan-view of the laminate. This approach limited their analysis to a rather coarse-grid model.

As shown in figure 1, polar symmetry can eliminate the need for complete plan-view models. The case presented in figure 1(a) is a simple $(\pm 45°)_s$ laminate with a circular hole and remote, uniaxial loading. The point $A(r, \theta)$ and its polar-symmetric counterpart $A'(r, \theta+\pi)$, by inspection, have the same displacements and stresses when expressed in polar coordinates. Accordingly, the displacements for the nodes in the shaded portion of figure 1(b) can be expressed in terms of their independent, polar-symmetric counterparts. This polar symmetry reduces the required modeling to one-fourth of the laminate, the unshaded portion of figure 1(b). Polar symmetry has been used previously in plate-bending and postbuckling analyses ([2] and [3], respectively), but the current study is believed to be its first application in a 3-D stress analysis.

To verify the present use of polar symmetry, a simple case was analyzed both with and without polar symmetry

(polar symmetry required only half as many nodes). The displacements and stresses were identical, as expected.

Although the present paper deals only with the simple $(\pm 45°)_s$ laminate, polar symmetry applies to the analysis of any laminate with angle plies. A finite-element computer code has been written to automatically exploit polar symmetry and is currently being used to analyze edge effects near holes. Because the new code requires only one-half as many nodes as needed previously, double the modeling refinement has been achieved for the same computing requirements.

# REFERENCES

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3. A. K. Noor, M. D. Mathers and M. S. Anderson, "Exploiting Symmetries for Efficient Postbuckling Analysis of Composite Plates," AIAA J., Vol. 15, No. 1 (January 1977), p. 24.

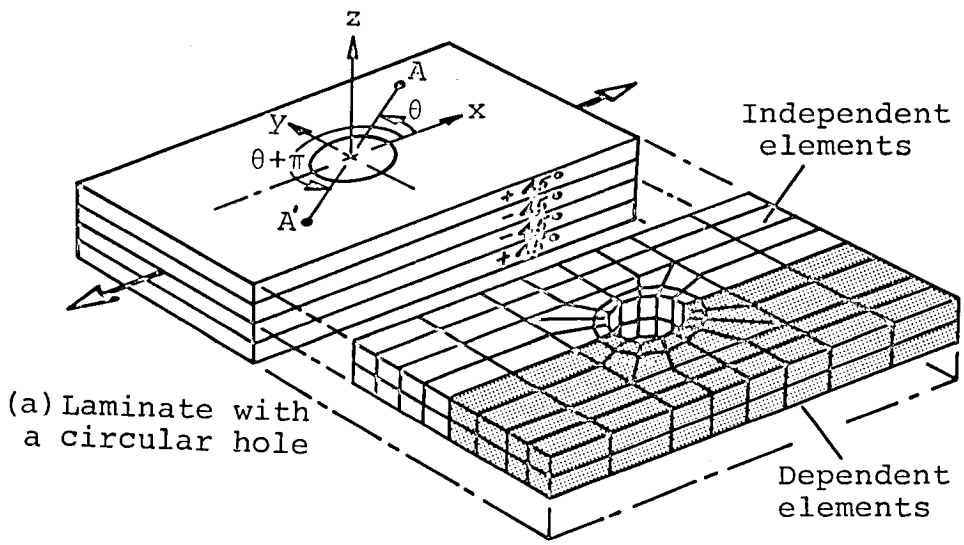


Figure 1. A composite laminate and its 3-D finite-element model using polar symmetry.

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15. Supplementary Notes

16. Abstract

This paper discusses an improved procedure for the 3-D finite-element analysis of an angle-ply laminate with a circular hole. The procedure exploits polar symmetry and is the basis for a new finite-element computer code. For the broad class of laminates that contain angle plies, this new code requires only one-half as much computing capacity as conventional codes. Because previous 3-D analyses have been limited by computer capacity, the new code will improve the analysis of laminates with angle plies.

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